# A Conceptual Framework for Energy Loss Minimization in Meshed MV Networks

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*Abstract*—Scope of this paper is to present a new conceptual framework for the optimal voltage regulation of active mediumvoltage networks with meshed topology. The proposed control concept coordinates the available reactive power of distributed generation (DG) units to efficiently tackle overvoltages, while also minimizing the daily energy losses. This is attained by developing a rule-based approach regarding the allocation of reactive power among the DG units, characterized by reduced computational complexity against the use of optimization techniques. The performance of the proposed concept is assessed and compared with respect to optimization-based centralized and decentralized control strategies by performing time-series simulations on the modified IEEE 123-bus distribution system.

*Index Terms*—Distributed power generation, loss minimization, reactive power control, voltage control.

## I. INTRODUCTION

Nowadays, distribution system operators (DSOs) are confronted with a new era regarding the secure and reliable network operation, imposed by the rapid increase of distributed generation (DG) units and especially of renewable energy sources. Considering European countries, this was mainly initiated by the Directive 2009/28/EC in an attempt to pave the way towards sustainability, flexibility, and efficiency [1]. Apart from the fact that the targets of this Directive have been revised upwards in the draft proposal of [2], there exist several problems DSOs must effectively address to allow the widespread use of renewable energy sources.

Voltage rise is considered as the most substantial technical issue [3], hindering the further increase of DG penetration in distribution networks. Focusing on medium-voltage (MV) networks, the most promising solution to this problem is to exploit the inherent ability of DG units to regulate network voltages by absorbing or providing reactive power, even during maximum generation conditions [4]. Recently, the research community proceeded a step further by treating this problem as an optimal voltage regulation problem, where the minimization of network losses constitutes an additional objective [5].

In the literature, the developed algorithms for the optimal voltage regulation can be classified into three main categories, namely decentralized, distributed, and centralized control schemes, presenting the following drawbacks:

*Decentralized:* This type of control schemes has been already incorporated in several grid codes for the interconnection of DG units by means of Q(V) and  $\cos \phi(P)$  droop characterictics [6]–[9]. Nevertheless, these methods cannot ensure the optimal network operation in terms of minimizing daily energy losses, since the control actions of each DG unit are determined based only on local measurements.

*Distributed*: These algorithms can be readily applied in distribution networks with radial and meshed topologies [10]. However, the convergence rate is highly dependent on the network size, prohibiting their implementation in real field conditions.

*Centralized*: This type of control strategies can be classified into two main subcategories based on whether they use optimization techniques or not. Nevertheless, the former is characterized by increased computational complexity and possible suboptimal solutions [11], [12], whereas the latter cannot ensure the optimal network operation. Additionally, the majority of these methods have been developed on the assumption of distribution networks with radial topology, limiting their applicability in meshed networks [13], [14].

This paper attempts to address the above-mentioned issues by developing a conceptual, rule-based framework for the optimal voltage regulation of meshed MV networks. The proposed control concept aims to regulate the network voltages within permissible limits by exploiting the reactive power capability of DG units, while also attaining minimum energy losses. Its distinct features include reduced computational complexity compared to optimization-based centralized algorithms, fast convergence rates against distributed algorithms, and near-optimal solutions compared to the decentralized control schemes.

The remaining of this paper is organized as follows: Sec-

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tion II presents the mathematical analysis of the optimal voltage regulation problem, whereas the proposed conceptual framework is thoroughly analyzed in Section III. Time-series simulations are performed in Section IV to assess the validity of the proposed concept. Finally, Section V concludes the paper.

## II. PROBLEM FORMULATION

The optimal voltage control, i.e., the regulation of network voltages in conjunction with the minimization of network losses, constitutes a nonlinear optimization problem, expressed mathematically as follows:

$$P_{\text{loss}} = \sum_{i \in N} \sum_{j \in N} [\alpha_{ij} (P_i P_j + Q_i Q_j) + \beta_{ij} (Q_i P_j - P_i Q_j)]$$
(1)

Eq. (1) is the objective function known as the exact loss formula [15], aiming to minimize the network power losses  $(P_{\text{loss}})$ . N is the set of network nodes, while  $P_i$  and  $Q_i$  denote the net injected active and reactive power at the *i*-th node, respectively. Moreover,  $\alpha_{ij}$  and  $\beta_{ij}$  are two coefficients calculated according to (2) and (3).

$$\alpha_{ij} = \frac{R_{ij}}{V_i V_j} \cos(\theta_i - \theta_j) \tag{2}$$

$$\beta_{ij} = \frac{R_{ij}}{V_i V_j} \sin(\theta_i - \theta_j) \tag{3}$$

Here,  $V_i$  and  $\theta_i$  denote the magnitude and angle of the complex voltage at *i*-th node, while  $R_{ij}$  is the real part of the *ij*-th element of the network Z-matrix. This matrix is actually the inverse of the network admittance matrix and can be directly calculated following the approach presented in [16]. The equality constraints of the optimization problem are the power flow equations, formulated according to:

$$P_i = V_i \sum_{j \in N} V_j (G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j))$$
(4)

$$Q_i = V_i \sum_{j \in N} V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j))$$
 (5)

where  $G_{ij}$  and  $B_{ij}$  stand for the real and imaginary part of the ij-th element of the network admittance matrix. It is worth noticing that the above mathematical formulation can be readily used for the analysis of distribution networks with meshed topology, compared to the use of the *DistFlow* equations that can be only applied to radial networks [17].

Furthermore, inequality constraints are introduced in the optimization problem to model the network operational limits in terms of permissible voltages as follows:

$$V_{\min} \le V_i \le V_{\max} \quad \forall i \in N \tag{6}$$

where  $V_{\rm min}$  and  $V_{\rm max}$  are the minimum and maximum permissible voltage limits, defined according to Standard EN50160 [18]. Finally, (7) is introduced to model the reactive power capability of the DG units.

$$Q_{\min,i} \le Q_i^{\mathrm{dg}} \le Q_{\max,i} \quad \forall i \in N_{\mathrm{dg}}$$
(7)

Here,  $N_{dg}$  is the set of network nodes where the DG units are connected.  $Q_i^{dg}$  is the control variable of the optimization problem, denoting the reactive power of the DG unit located at the *i*-th node, whereas  $Q_{\min,i}$  and  $Q_{\max,i}$  are the corresponding permissible limits, defined by the reactive power capability of the DG unit [19].

Exact solution to the optimization problem of (1)-(7) requires increased computational resources, preventing its implementation for the real-time operation of extended distribution networks. Moreover, several research attempts have failed to effectively solve this optimization problem in meshed networks, main due to the increased computational complexity introduced by their complex structure [14].

#### III. CONCEPTUAL FRAMEWORK

Scope of the proposed methodology is to reduce the computational complexity introduced by the meshed topology and the use of optimization techniques in order to allow the real-time voltage regulation of distribution networks with near-minimum network losses. The proposed conceptual framework consists of a rule-based approach for the reallocation of reactive power among the DG units, which can be summarized in the following statement:

Overvoltage mitigation with minimum losses can be achieved if the reactive power absorption process is undertaken only by the DG unit presenting the maximum network voltage at the point of common coupling (PCC).

The validity of above-mentioned statement has been mathematically proven in a previous work assuming radial networks [20]. In this paper, its applicability is further investigated in distribution networks with meshed topology. Considering a given time instant with constant generation and consumption data, the proposed reactive power allocation process is depicted in Fig. 1 by means of a flowchart, consisting of the following 7 steps:

**Step 1:** *Voltage acquisition.* Initially, the network voltages are acquired. In case of implementing the proposed methodology in real field conditions, this can be attained using monitoring or state estimation methods.

**Step 2:** *DG unit selection.* In this Step, the DG unit located at the *i*-th node that presents the maximum PCC voltage  $(V_i^{\max})$  is selected to undertake the reactive power absorption process in order to mitigate potential overvoltages.

**Step 3:** *Check for overvoltage violation.* In case an overvoltage violation occurs, the procedure moves to Step 4. Otherwise, the reactive power allocation process is terminated.

**Step 4:** Overvoltage mitigation process. In such a case, the DG unit determined according to Step 2, increases incrementally the absorbed reactive power to compensate the voltage rise caused by the corresponding active power injection.

**Step 5:** *Voltage acquisition*. In this Step, the new network voltages are acquired.



Fig. 1. Flowchart for overvoltage mitigation with minimum losses.

**Step 6:** *Check for overvoltage mitigation.* In case the overvoltage at the PCC of the selected DG unit has been successfully tackled, the voltage regulation of the network is accomplished and the procedure is terminated. Otherwise, it moves to Step 7.

**Step 7:** Check for selecting another DG unit. In this Step, the PCC voltage of the selected DG unit, i.e.,  $V_i^{\max}$ , is compared with the PCC voltages of the remaining DG units denoted by the vector  $V_{dg}$ . If  $V_i^{\max}$  is greater than  $V_{dg}$ , the procedure moves to Step 4 and the reactive power absorption of the selected DG unit is further increased. Otherwise, the procedure moves to Step 2 and a new DG unit is selected to participate in the reactive power allocation process.

#### **IV. SIMULATION RESULTS**

The effectiveness of the proposed conceptual framework is demonstrated on the modified IEEE-123 bus distribution system of Fig. 2, while its performance is compared with wellestablished decentralized and centralized control schemes. The examined MV network has been widely used in the literature as a benchmark meshed network for the control and analysis of power systems. In this network, only one type of DG units is assumed, namely photovoltaics (PVs), with a rated power factor equal to 0.85, whereas the voltage at the slack bus is considered equal to 1.05 p.u. to achieve overvoltage violations above the maximum permissible limit of 1.1 p.u. Further details regarding the network technical characteristics can be found in [21].



Fig. 2. Network topology.

In the next subsections, the basic configuration of the decentralized and centralized control schemes is firstly presented. Afterward, the key features of the parametric analysis are analytically described. Finally, the corresponding numerical results are demonstrated.

#### A. Examined Control Schemes

The proposed methodology is compared with two decentralized algorithms, namely the Q(V) and the  $\cos \phi(P)$  droop control strategies. In the former, the reactive power absorption of the each PV is calculated with respect to the PCC voltage [6], [7], and the maximum reactive power absorption depends on the reactive power capability of the PV. In this paper, the voltage threshold for the activation of the droop control is considered equal to 1.08 p.u., as defined in the Italian grid code [8]. In the latter, the power factor of the PV depends on the injected active power [7] with a minimum value equal to 0.95, and the corresponding power threshold is set equal to 50% of the rated power [9]. Additionally, the proposed method is compared with an optimization-based centralized algorithm, which is modeled using (1)-(7) and solved in GAMS using the BONMIN solver [22]. To achieve a common comparable basis among the examined control schemes, it is assumed that PV units can only absorb reactive power.

## B. Configuration of the Performed Analysis

Time-series simulations are performed on the system under study depicted in Fig. 2. The simulation period is one day with one minute time interval. The normalized daily consumption profiles of Fig. 3 are arbitrarily distributed among the network loads. Considering PV generation, the overall installed capacity is equal to 20 MWp, whereas a sunny day with clear sky is considered to examine the worst case scenario in terms of reverse power flows and thus of overvoltages.

To demonstrate the generic capability of the proposed methodology towards optimality and efficiency, a parametric analysis is performed. In this analysis, a set of different system



Fig. 3. Typical daily consumption profiles.

configurations are created by varying the number, the location, and the installed capacity of the PVs. More specifically, 6 test cases are considered regarding the number of PVs, varying from 10 to 60. In each test case, 15 PV allocation patterns and 15 rated power distribution patterns are arbitrarily created, resulting in 225 different combinations. Therefore, the conducted parametric analysis consists of 1350 different system configurations.

#### C. Numerical Results

Considering the first test case of 10 PVs, the corresponding numerical results are presented in Figs. 4-7. In particular, the overall reactive power consumption of the PVs and the daily network voltages are depicted in Fig. 4 and Fig. 5, respectively, for all the examined control schemes. The blue color is employed to represent the variation range in each time instant, caused by the 225 examined combinations regarding the location and the installed capacity of the PVs. Additionally, the network daily losses for all the 225 combinations and the first 15 combinations are depicted in Fig. 6 and Fig. 7, respectively.

By employing the  $\cos \phi(P)$  droop control strategy, the overall reactive power consumption of the PVs is significantly increased compared to the other examined methodologies, as shown in Fig. 4. The main reason lies in the fact that the reactive power absorption in each PV is calculated with respect to the injected active power, neglecting the current condition of the network voltages. This uncoordinated operation leads to a high reduction of network voltages below the minimum permissible limit of 0.9 p.u., as verified in Fig. 5. Furthermore, since the overall installed capacity is kept constant to 20 MWp and the same generation profile is applied to the PVs, the unique reactive power profile of Fig. 4 applies for all the examined 225 combinations regarding the location and the installed capacity of the PVs. Considering network losses, this excessive and unnecessarily high reactive power consumption leads to remarkably increased daily energy losses, as observed in Fig. 6.

The situation is improved when the Q(V) droop control strategy is applied. More specifically, according to Fig. 4, it can be observed that the overall reactive power consumption



Fig. 4. Overall reactive power of PVs. (a)  $\cos \phi(P)$ , (b) Q(V), (c) OPF, and (d) proposed methodology.



Fig. 5. Daily network voltage profiles. (a)  $\cos \phi(P)$ , (b) Q(V), (c) OPF, and (d) proposed methodology.

is significantly reduced compared to the  $\cos \phi(P)$  method, leading also to reduced daily energy losses, as shown in Fig. 6 and Fig. 7. This happens due to the fact that the network condition is indirectly taken into account by introducing the PCC voltage as the control variable for the calculation of the reactive power in each PV. Thus, high amounts of consumed reactive power during high generation periods are effectively avoided. Nevertheless, the Q(V) droop control method presents increased overall reactive power consumption, and thus network losses, compared to the OPF method and the proposed conceptual framework. The main reason lies in the existence of the droop control mechanism which is activated in voltages below the maximum permissible limit of 1.1 p.u., resulting in unnecessarily reactive power absorption and the bell-shape maximum voltage profile of Fig. 5. Finally, the network voltages are effectively regulated within permissible limits in the Q(V) implementation, as shown in Fig. 5.

In the OPF control strategy, the network operational limits are fully exploited to minimize the network daily losses. More specifically, as shown in Fig. 5, the maximum network voltages are equal to the upper permissible limit of 1.1 p.u. to minimize the required amount of reactive power absorption, during high generation periods. This is also verified in Fig. 4 and Fig. 6, where the OPF method presents reduced overall reactive power consumption and network losses, compared to decentralized



Fig. 6. Network daily energy losses of the first test case (10 PVs).



DAILY ENERGY LOSSES (MWH)					
PV number	Scheme	Min	Max	Mean	Std
10	$\cos\phi(P)$	12.47	25.79	17.29	3.07
	Q(V)	8.87	17.37	12.09	1.85
	OPF	8.65	16.87	11.78	1.81
	Proposed	8.65	16.88	11.78	1.81
20	$\cos \phi(P)$	9.74	17.98	14.16	1.89
	Q(V)	7.00	12.45	10.00	1.34
	OPF	6.85	12.11	9.81	1.21
	Proposed	6.85	12.11	9.81	1.21
30	$\cos \phi(P)$	9.95	18.99	14.67	1.98
	Q(V)	7.23	13.09	10.18	1.21
	OPF	6.98	12.78	10.11	1.26
	Proposed	6.98	12.78	10.18	1.27
40	$\cos \phi(P)$	11.56	16.90	14.63	1.32
	Q(V)	8.22	11.79	10.16	0.92
	OPF	8.05	11.54	10.06	0.85
	Proposed	8.05	11.54	10.06	0.85
50	$\cos \phi(P)$	13.03	17.93	14.74	1.00
	Q(V)	9.23	12.41	10.42	0.76
	OPF	8.97	12.08	10.14	0.62
	Proposed	9.02	12.08	10.15	0.62
60	$\cos\phi(P)$	11.87	17.43	15.21	1.21
	Q(V)	8.46	12.23	10.52	0.87
	OPF	8.29	12.13	10.52	0.80
	Proposed	8.29	12.13	10.52	0.80

TABLEI

Fig. 7. Network daily energy losses of the first test case (10 PVs) with respect to the distribution of rated power among the PVs for a given allocation pattern.

control schemes. Nevertheless, the main drawback of this method is the increased computational complexity and possible sub-optimal solutions, limiting its real-time implementation in extended distribution networks with meshed topology.

The proposed conceptual framework presents a similar performance to the OPF method. This can be verified in Fig. 6 and Fig. 7, where the corresponding curves of daily energy losses are overlapped. Moreover, in the proposed method, the network voltages are kept within permissible limits, as shown in Fig. 5, while the reactive power profile is similar to corresponding derived by the OPF method. Consequently, the proposed conceptual framework can be readily applied in distribution meshed networks, presenting improved performance compared to the decentralized methods and low computational complexity against the OPF control strategy.

In Fig. 6, the location of the PVs changes every 15 combinations, i.e., 15 different distributions of installed capacity among the PVs. Therefore, it can be observed that the network daily energy losses are strongly dependent on the location of the PVs along the network and they are less dependent on the distribution of installed capacities. Nevertheless, in all the examined combinations, the proposed method outperforms compared to the decentralized droop control strategies and the centralized OPF-based method.

Finally, a statistical analysis is conducted concerning the simulation results derived from the 6 examined test cases,

where the number of installed PVs varies between 10 and 60. In particular, Table I presents the minimum, maximum, average, and the standard deviation of the network daily energy losses for all the examined control schemes. It can be concluded, that the proposed methods presents a superior performance compared to the decentralized strategies and a similar performance to the OPF control scheme. Thus, this parametric analysis verifies the generic capability of the proposed method to achieve near-optimal solutions with reduced computational complexity.

## V. CONCLUSIONS AND FUTURE WORK

In this paper, a new conceptual framework is proposed for the optimal voltage regulation of distribution networks with meshed topology by exploiting the reactive power capability of DG units. The proposed framework adopts a rule-based approach regarding the optimal distribution of reactive power consumption among the DG units to effectively tackle overvoltages, while also minimizing network losses.

To thoroughly evaluate the performance of the proposed conceptual framework against well-established decentralized and centralized OPF-based methods, a parametric analysis is performed by varying the number, the location, and the installed capacity of the DG units. Simulation results revealed that the proposed method presents a superior performance compared to the examined control schemes. More specifically, the proposed conceptual framework can achieve a near-optimal solution compared to the decentralized control methods, while maintaining low computational complexity against the OPF method. Therefore, it can be a valuable solution for DSOs towards the increased penetration of DG in distribution networks with meshed topology.

Further work will be carried out concerning the real-time implementation of the proposed conceptual framework. In this way, the applicability of the proposed method in real field conditions will be further investigated.

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